Magnus Effect on Spinning Bodies of Revolution

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Theme

THIS paper presents an analytical theory for the prediction of Magnus sideforce on spinning bodies of revolution at small angles of attack. Previous theories predict a linear variation of sideforce coefficient with angle of attack α. Available wind-tunnel data invariably shows a nonlinear variation, with the previous theories being accurate only in the limit of zero α. The authors attempted to improve this situation by assuming an axial circulation distribution along the spinning body axis using a cross flow analogy. Such circulation is the result of the separation of the boundary layer that causes a shedding of vorticity into the outer flow. The experimental data imply that this shedding is physically present even at small α. Any small α theory based upon the solution of a boundary-layer equation formulation (as is the present theory) would be strictly applicable only, at best, to the growth of the boundary layer before its separation and the actual details of the crossfield separations are thus ignored. It has been found, however, that use of the crossflow analogy leads to a theory which correlates very well with experimental results. In fact, even though the resulting theory is subject to the previously discussed limitations, it has shown a remarkable accuracy at large angles of attack.

Contents

Boundary-layer theory is used to study the exterior flow about an open-end rotating cylinder. A nonrotating cylindrical coordinate system is used with a uniform freestream velocity U inclined at a small angle-of-attack α and the cylinder with radius r_0 is to be rotating at a small nondimensional roll rate $\bar{p} = r_0\omega/U$ where ω is the spin angular velocity. A perturbation solution about $\alpha = \bar{p} = 0$ is found.

The boundary conditions at the cylinder surface are clearly

$$u = v = 0, \qquad w = \bar{p}U \tag{1}$$

where u, v, and w are the flow velocity components in the x, r, and ϕ (axial, radial, lateral) directions. However, the conditions at the outer edge of the boundary layer require more discussion. Both Martin[1] and Kelly[2] assumed that no circulation was present in the inviscid outer flow. This is certainly true for very small angles of attack. It has been established that there is indeed a circulation at higher angles of attack caused by the separation of the boundary layer. The resulting shed vorticity induces an equal but opposite circulation in the outer flow. It is through this mechanism, at angles of attack sufficiently large for vorticity to be shed, that the moving wall communicates its effects to the outer flow. It was assumed that this circulation could be approximated by assuming that the crossflow along the x axis of the spinning cylinder at low angles of attack is similar to the unsteady solution of the flow over an impulsively rotated cylinder placed perpendicular to the freestream. This leads to an axial circulation distribution for the spinning missile. The boundary conditions at the outer edge were assumed to be similar to those of a slender body of revolution at an angle of attack when acted upon by the superposition of a uniform freestream velocity U and an axial irrotational vortex distribution $\Gamma(x)$. The resulting boundary conditions are

$$\begin{aligned} u &= U\cos\alpha \simeq U[1-(\alpha^2/2)] \\ w &= U\alpha\sin\Phi[1+(R_0{}^2/r^2)] + k(x)\Gamma_0/2\pi r \end{aligned} \tag{2}$$

where $\Gamma_0 = 2\pi r_0 V$, $k(x)$ is the fraction of Γ_0 that corresponds to the inviscid circulation $\Gamma(x)$, and R_0 is the distance from the cylinder axis to the outer edge of the boundary layer. The velocities are assumed to be in the form

$$u/U = u_0 + u_1 + u_2; \quad v/U = v_0 + v_1 + v_2; \quad w/U = w_1 + w_2 \tag{3}$$

where the subscript corresponds to the order of the small perturbation quantities α and $\bar{p}$. Since at $\alpha = 0$, u and v are even functions of $\bar{p}$ and w is an odd function, u_1 and v_1 are independent of $\bar{p}$, and w_2 is independent of $\bar{p}^2$.

Each order of the assumed velocity is then substituted into the equation of motion using the normal dimensionless boundary-layer coordinates

$$\xi = (4/r_0)(vx/U)^{1/2} \tag{4}$$

$$\eta = [(r^2 - r_0{}^2)/4r_0](U/vx)^{1/2} \tag{5}$$

The resulting equations are then reduced to a system of ordinary differential equations which were solved numberically by digital computer.[3]

This system is sufficient to determine the boundary-layer velocity profiles if the circulation distribution $k(x)$ is known. Values of the time-dependent lift coefficient, estimated from Thoman and Szewczyk's results,[4] correlate very well for small values of time by the relationship

$$C_L \sim (U_\infty t/d)(V/U_\infty)[(U_\infty d/v)^{1/4}] \tag{6}$$

where V is the surface speed of the cylinder. To apply this result to the spinning cylinder crossflow let

$$U_\infty = U\sin\alpha; \quad t = x/U\cos\alpha; \quad V/V_\infty = \bar{p}/\sin\alpha \tag{7}$$

Since the local circulation strength is

$$\Gamma = \tfrac{1}{2}C_L U_\infty d$$

the circulation function is

$$k = \frac{\Gamma}{\Gamma_0} = \frac{C_0}{2\pi}\left(\frac{x}{d}\right)\tan\alpha \bigg/ \left(\frac{Ud\sin\alpha}{v}\right)^{1/4} \tag{8}$$

Assuming small α and transforming to the dimensionless coordinates

$$k \simeq (C_0/128\pi)R_C{}^{3/4}\xi^2 \tag{9}$$

where R_C is the crossflow Reynolds number based on the cylinder diameter. Because the circulation is of order ξ^2 for reasonable

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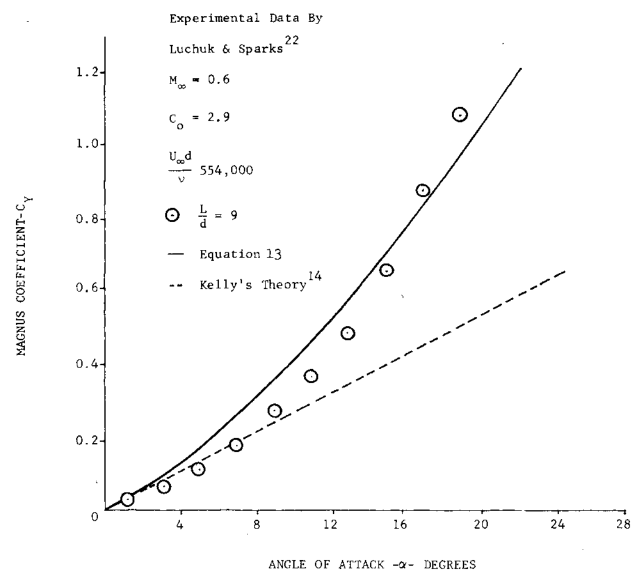

Fig. 1 Sideforce coefficient comparison, $M_\infty = 0.6$.

Reynolds numbers most of the boundary-layer profile functions are affected very little by the assumed circulation when compared to Kelly's.

The calculation of the Magnus sideforce involves contributions due to the displacement thickness, the radial pressure gradient, skin friction, and the circulation distribution. The displacement thickness calculation involves an imaginary warped cylindrical body generated by adding the boundary-layer displacement thickness to the cylinder. The solution for an inviscid flow about this new surface results in the appearance of a sideforce, the displacement thickness contribution to the total Magnus force. The displacement thickness effect is calculated assuming that the pressure throughout the boundary layer is constant in the radial direction. The radial pressure gradient contribution takes into account the pressure change from the outer edge of the boundary layer to the cylinder surface. The surface shearing stress is an asymmetric function of the angle Φ for the rotating cylinder. Integration of this shearing stress over the cylinder surface provides another contribution to the total Magnus force. The final contribution is that due to the axial circulation distribution caused by vorticity shed into the crossflow wake. The local sideforce is related to the local circulation strength by Rayleigh's theory for lift on a rotating circular cylinder placed normal to the freestream. The total Magnus force is obtained by integration of all the local sideforce contributions over the length of the cylinder.

The contributions to the Magnus force due to displacement thickness, radial pressure gradient, and skin friction were calculated to be essentially those predicted by Kelly. The contribution to the Magnus force coefficient due to the circulation is calculated by

$$Y_4 = \int_0^L \rho U \alpha k(x) \Gamma_0 \, dx \tag{10}$$

Since the circulation for a given α and $\bar{p}$ was found to be in the form

$$k = C_1 R_C{}^{3/4} \xi^2 \tag{11}$$

(where C_1 is a constant) the Magnus sideforce coefficient due to the circulation distribution is

$$C_{Y_4} = (256 C_1 R_C{}^{3/4}/R_L)(L/d)^3 \alpha \bar{p} \tag{12}$$

The total Magnus force coefficient is the sum of these contributions and is

$$C_Y = \frac{\alpha \bar{p}}{R_L{}^{1/2}}\left(\frac{L}{d}\right)^2 \left\{ 60.97 + (256 C_1 R_C{}^{3/4} - 357.37)\left(\frac{L}{d}\right)\frac{1}{R_L{}^{1/2}} - 227.01\left(\frac{L}{d}\right)^2 \frac{1}{R_L} \right\} \tag{13}$$

Note that the term in Eq. (13) involving the circulation is nonlinear in angle of attack, contrary to previous boundary-layer analyses.

Figures 1 and 2 show comparisons of the present result, Kelly's theory and wind-tunnel data from Luchuk and Sparks. The addition of a circulation distribution to the theory greatly improves Kelly's result for higher α's. In fact the wind-tunnel data shape is accurately predicted for angles much higher than could be expected. The assumption of small α and a thin boundary layer must certainly be violated at these higher angles of attack. From this result it is concluded that at the higher angles of attack the dominant mechanism for the development of sideforce is vorticity shedding and that boundary-layer contributions are small.

Up to approximately 15° the sideforce coefficient magnitude and shape are predicted fairly well by Eq. (13). No attempt has been made to correct the theory for the effects of a nose shape placed in front of the cylinder. All wind-tunnel data to date has been taken using a body of this type. Accurate sideforce data is extremely difficult to obtain since the sideforce magnitude is much smaller than the other forces acting upon the cylinder. Often quite different results for experimental Magnus force coefficients are obtained between tests run at positive and negative spin rates. Varying nose shape or testing in different wind tunnels can also cause large effects of the experimental results. Considering these problems, the essential nonlinearity of the Magnus force coefficient is well represented by this theory.

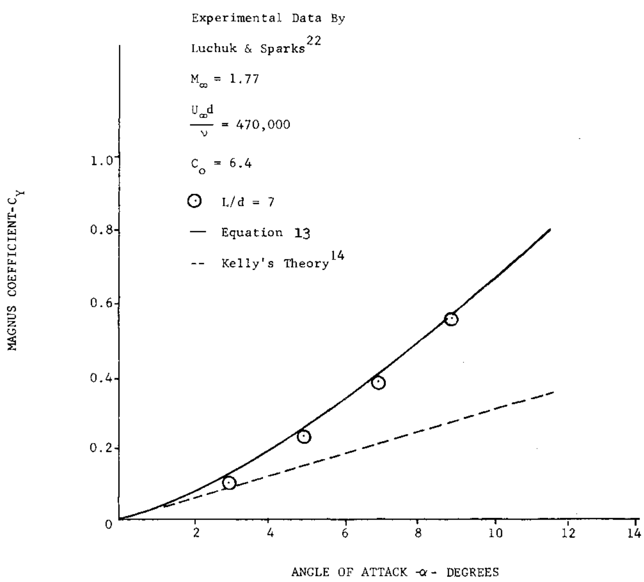

Fig. 2 Sideforce coefficient comparison, $M_\infty = 1.77$.

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